

Analysis of the Uncertainty Propagation through Fitting Spectral Irradiance Data

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Abstract. Uncertainties and correlations are propagated in absolute spectral irradiance scale, where a modified Planck's radiation law is fitted to filter radiometer (FR) data. The lack-of-fit error component is included in the uncertainty analysis with the purpose of finding an optimal degree for the polynomial describing the effective emissivity of quartz-tungsten halogen lamps.

Introduction

Absolute scales of spectral irradiance are widely established using detector-based facilities [1, 2, 3, 4, 5, 6]. In general, such scales are realized at discrete wavelengths while the irradiance over the entire spectral range is obtained by interpolation or fitting, as suggested for example in [7]. It has been shown recently that rigorous uncertainty analysis for the derived spectral irradiance values requires correlations through fitting and those in the FR data to be determined and included in the propagation of uncertainties and covariances [8, 9, 10]. This is also required by the internationally agreed rules for expressing uncertainties in measurements [11].

In our earlier report we presented analysis of the propagation of the uncertainties and covariances in the spectral irradiance scale where the lamps are measured directly, without the use of a black body radiator [10]. The study estimated the effect of the correlations in the FR data on the fitted irradiance values. The other useful finding of the uncertainty analysis including correlations between the fitted parameters was that it could be useful for selecting the FR wavelengths.

In this contribution, we expand the study to include the lack-of-fit component in the uncertainty of the fitted values. The lack-of-fit error and the propagated uncertainty are both depended on the description of the effective emissivity of the lamp. The purpose of the analysis is to find an optimal degree for the emissivity polynomial such that the observed overshoots in the output uncertainty between the FR wavelengths are reduced while keeping the lack-of-fit error reasonably small.

Propagation of Uncertainties in FR Data

With the purpose of studying the propagation of uncertainties and correlations through the spectral irradiance scale, where a modified Planck's radiation law is fitted to a full range of FR data points, we applied the matrix formalism by Woeger [10, 12]. In the modified Planck's formula being fitted, an N^{th} -degree polynomial was employed for the description of the effective emissivity of quartz-tungsten halogen lamps [5, 10]. The coefficients of the polynomial were determined during the

fitting.

Results of the Analysis

The uncertainties of the spectral irradiance propagated from those in the FR measurements (input uncertainty) of a 1-kW FEL-type lamp (same as the one analyzed in [10]) through the fitting are plotted in Figure 1. Here, for demonstrational purposes the FR data point at 850 nm was excluded from the analysis. From the figure it would appear that the uncertainties in the fitted spectral irradiance values could be reduced by decreasing the order of the emissivity polynomial. However, as can be seen from Figure 2 the decrease in the order of the polynomial has an opposite effect on the lack-of-fit error. The increase in the lack-of-fit error is especially pronounced at the lower wavelengths where the effective emissivity of tungsten lamps has a complex structure [5]. It can be also noticed from Figure 1 and Figure 2 that the use of higher-order emissivity polynomial although reduces the lack-of-fit error, but at the same time it may cause overshoots in the uncertainty between the FR wavelengths. It has to be noted, however, that the presence of strong correlations in FR data would lead to a lower level of oscillations in the propagated uncertainty [9, 10]. Having correlation coefficients of 0.5 between all wavelengths reduces the amplitude of the uncertainty oscillations at longer wavelengths as shown in Figure 1 by almost one third.

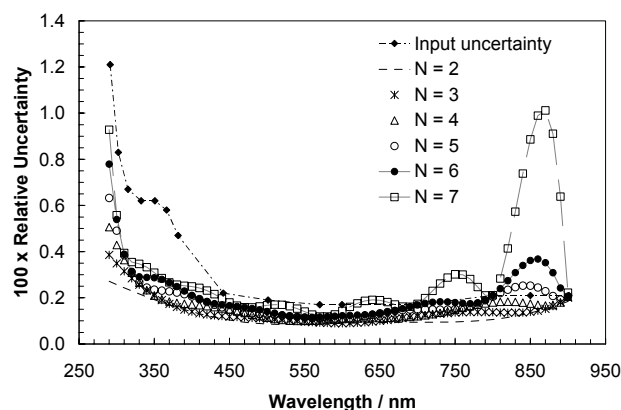


Figure 1. Uncertainty propagation through fitting the Planck's radiation law modified by N^{th} -degree emissivity polynomial to the FR data. The input uncertainty is the uncertainty in the FR measurements at discrete wavelengths. The output uncertainties denoted as $N=2, \dots, N=7$ were calculated for 290 to 900 nm wavelengths with a 10 nm step and no correlations between the FR-measured values.

In Figure 3, the uncertainty component due to the lack-of-fit error is shown together with the uncertainty propagated from the FR measurements when the lack-of-fit uncertainty is and is not included. The lack-of-fit

uncertainty component was estimated as a standard deviation of the differences between the fitted and measured spectral irradiance values at the FR wavelengths. The combined uncertainty was calculated by quadratically adding the propagated uncertainty and that due to the lack-of-fit error. The uncertainties depicted in Figure 3 represent average of the values calculated every 10 nm within 290 to 900 nm spectral range. As can be seen from the figure, if average uncertainties over the whole spectral range are considered the selected 6th-degree polynomial is an optimal choice for the description of the effective emissivity of the tungsten lamp. However, this may not necessarily be true if individual wavelengths are considered, such as those within 800 to 900 nm range where an overshoot in the propagated uncertainty is caused by the high order of the polynomial. It can be also noticed from the figure that the 6th-degree polynomial remains still an optimal choice even if moderate correlations in the FR data are considered.

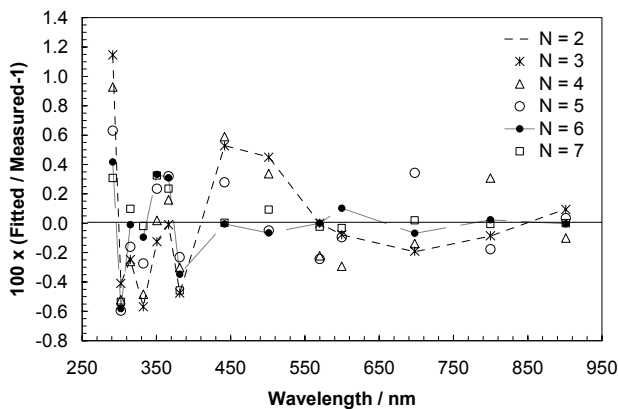


Figure 2. Relative difference between the measured and fitted spectral irradiance values when the effective emissivity is modeled by N^{th} degree polynomial.

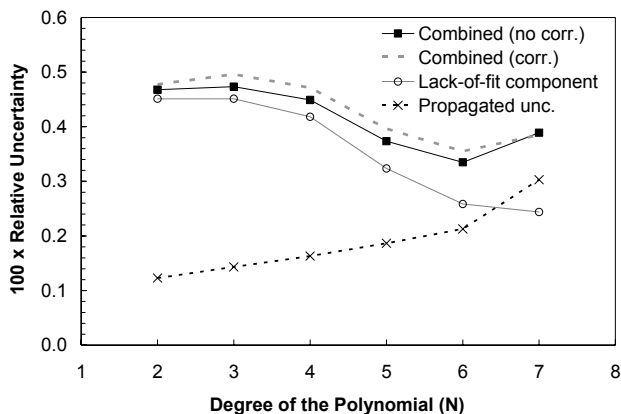


Figure 3. The lack-of-fit uncertainty component (open circles), the uncertainty propagated from the FR-measurements without correlations at the input values (crosses), and their quadratic sum (filled squares). The uncertainty denoted as combined correlated (dashed line) is the result with a correlation coefficient of 0.5 in the FR data. The uncertainties are plotted as a function of the degree of the emissivity polynomial. The shown values are averages of all the values calculated over 290 to 900 nm wavelengths with a 10 nm step.

Conclusions

We propagated the uncertainties and correlations in the primary spectral irradiance scale where the irradiance of a lamp is obtained from direct measurements with filter radiometers (FRs). The lack-of-fit component was also included in the uncertainty calculations. The effect of the degree of the polynomial describing the effective emissivity of the lamp on the lack-of-fit error and on the propagated uncertainty values was studied with the purpose of finding an optimal value. The mathematically optimal degree of the emissivity polynomial was found to remain the same even in the case of moderate correlations in the FR data.

References

1. Boivin L.P., Gaertner A.A., Analysis of the uncertainties involved in the realization of a spectral irradiance scale in the infrared at the NRC, *Metrologia*, 28, 129-134, 1991.
2. Anderson V.E., Fox N.P., A new detector-based spectral emission scale, *Metrologia*, 28, 135-139, 1991.
3. Johnson B.C., Cromer C.L., Saunders R.D., Eppeldauer G., Fowler J., Sapritsky V.I., Dezsi G.A., A method of realizing spectral irradiance based on an absolute cryogenic radiometer, *Metrologia*, 30, 309-315, 1993.
4. Sperfeld P., Raatz K.-H., Nawo B., Möller W., Metzdorf J., Spectral-irradiance scale based on radiometric black-body temperature measurements, *Metrologia*, 32, 435-439, 1995.
5. Kübarsepp T., Kärhä P., Manoocheri F., Nevas S., Ylianttila L., Ikonen E., Spectral irradiance measurements of tungsten lamps with filter radiometers in the spectral range 290 nm to 900 nm, *Metrologia*, 37, 305-312, 2000.
6. Yoon H. W., Gibson C. E., Barnes P. Y., Realization of the National Institute of Standards and Technology detector-based spectral irradiance scale, *Appl. Opt.*, 41, 5879-5890, 2002.
7. Walker J. H., Saunders R. D., Jackson J. K., McSparron D. A., NBS measurement services: spectral irradiance calibrations, *Natl. Bur. Stand. Spec. Publ.*, 250-20, 1987.
8. Gardner J. L., Correlations in primary spectral standards, *Metrologia*, 40, S167-S171, 2003.
9. Gardner J. L., Uncertainty propagation for NIST visible spectral standards, *J. Res. NIST*, 109, 306-318, 2004.
10. Nevas S., Ikonen E., Kärhä P., and Kübarsepp T., Effect of correlations in fitting spectral irradiance data, *Metrologia*, 41, 246-250, 2004.
11. Guide to the Expression of Uncertainty in Measurement, 101 p., *International Organization for Standardization*, Geneva, 1993.
12. Woeger W., Uncertainties in models with more than one output quantity, in *CIE Proceedings of the CIE Expert Symposium 2001*, pp. 12-17, CIE, Vienna, 2001.