On Estimation of Distribution Temperature

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Abstract. Different least square estimation methods for determination of distribution temperature are compared. Their applicability is discussed in regard to uncertainties of spectral irradiance data of incandescent lamps. Experimental results are analyzed to support theoretical conclusions.

Introduction

Radiation from an incandescent lamp, especially a tungsten filament lamp, has a relative spectral power distribution which is reasonably close to that of a blackbody radiator. Therefore, it is often convenient to describe incandescent radiation in terms of the temperature of an equivalent Planckian radiator. Corresponding to CIE recommendations [1] the distribution temperature T_D of a source in a given wavelength range (λ_1 to λ_2) is obtained by minimizing the following sum with regard to the parameters a, T:

$$\sum_{\lambda_i=\lambda_1}^{\lambda_2} \left[1 - E(\lambda_i)/aP(\lambda_i, T)\right]^2 \qquad (1)$$

$E(\lambda_i)$ relative spectral irradiance of the source at wavelength λ_i

$P(\lambda_i,T)$ relative spectral irradiance of a blackbody radiator at temperature T and wavelength λ_i

$$P(\lambda_i, T) = \lambda_i^{-5}\left[\exp(c_2/\lambda_i T) - 1\right]^{-1} \qquad (2)$$

$c_2 = 1.4388 \cdot 10^{-2}$ mK (second radiant constant)

a scaling factor

Summation over wavelengths with equidistant intervals of maximum $(\lambda_{i+1}-\lambda_i)$ = 10 nm are recommended in photometry in the spectral range from λ_1 = 400 nm to λ_2 = 750 nm [1].

Least Squares Estimation

The concept of least squares estimation is generally based on the following model:

$$E(\lambda_i) + \varepsilon(\lambda_i) = aP(\lambda_i, T) \qquad (3)$$

$\varepsilon(\lambda_i)$ random error with:

expectation value zero	$E[\varepsilon(\lambda_i)] = 0, \forall \lambda_i$,
no correlation	$E[\varepsilon(\lambda_i)\varepsilon(\lambda_k)] = 0, \forall \lambda_i \neq \lambda_k$,
constant variance	$E[\varepsilon^2(\lambda_i)] = \sigma^2, \forall \lambda_i$.

For spectral irradiance data sets $\varepsilon(\lambda_i)$ is usually in proportion to the value of spectral irradiance $E(\lambda_i)$:

$$\varepsilon(\lambda_i) = E(\lambda_i)\eta(\lambda_i) \qquad (4)$$

Thus the variance of ε is not constant for all λ_i, but the variance of η may be considered as constant over a limited wavelength region. This leads to:

$$E(\lambda_i)\left(1 + \eta(\lambda_i)\right) = aP(\lambda_i, T) \qquad (5)$$

To apply least squares techniques on this model, equation (5) is logarithmised and simplified (Taylor series):

$$\ln\left(E(\lambda_i)\right) + \eta(\lambda_i) \approx \ln\left(aP(\lambda_i, T)\right) \qquad (6)$$

Thus, for estimation of a and T the following sum is minimized:

$$\sum_{\lambda_i=\lambda_1}^{\lambda_2} \left[\ln\left(E(\lambda_i)/aP(\lambda_i, T)\right)\right]^2 \qquad (7)$$

Since the values of the argument of the logarithm will tend to 1 due to the minimization process, (7) can be simplified to (Taylor series):

$$\sum_{\lambda_i=\lambda_1}^{\lambda_2} \left[1 - E(\lambda_i)/aP(\lambda_i, T)\right]^2 \qquad (8)$$

This resembles formula (1). Therefore, the estimation method recommended by CIE is designed for errors, which are in proportion to the corresponding value of spectral irradiance. A numerical solution of the minimization problem (8) is provided in [2].

If the condition of homoscedasticity for least squares estimation (i. e. the variance of ε is constant for all λ_i) is not fulfilled, usually the method of weighted least squares is applied:

$$\sum_{\lambda_i=\lambda_1}^{\lambda_2} \left[E(\lambda_i) - aP(\lambda_i, T)\right]^2 / u^2\left(E(\lambda_i)\right) \qquad (9)$$

It seems, that this estimation method is already well in use to determine T_D [3]. A numerical solution for the minimization problem (9) is also given in [3]. If the uncertainties are in proportion to $E(\lambda_i)$ (i. e. $u(E(\lambda_i)) = u_{rel} \cdot E(\lambda_i)$) and the relative uncertainty u_{rel} is constant over the wavelength range from λ_1 to λ_2, the CIE estimation method (8) and weighted least squares (9) should provide similar results for T_D. Based on the results in [4] a Monte Carlo simulation was developed to generate realistic spectra of an incandescent lamp at 2856 K (CIE A). The wavelengths range from λ_1 = 400 nm to λ_2 = 750 nm with wavelength intervals of 5 nm. u_{rel} was chosen in the range of 0.5% to 1.5%. Differences between the two estimators for T_D obtained by minimizing (8) and (9), respectively, did not exceed 0.8 K and are therefore negligible. In addition, the spectrum of the BEV detector-based FEL-lamp (irradiance standard) was generated employing natural cubic splines based on calibration data provided by PTB (Physikalisch-Technische Bundesanstalt). The relative uncertainties of spectral irradiance data in the wavelength range from 400 nm to 750 nm were 0.8%. The wavelength interval was again 5 nm. The results of the two estimation methods (8),

(9) were identical (T_D = 3112.7 K). This confirms, that the estimation method recommended by CIE and weighted least squares with constant u_{rel} provide statistically equal results. Further simulations will be presented.

The use of the CIE method weighted with u_{rel} to determine T_D is suggested in [5]:

$$\sum_{\lambda_i=\lambda_1}^{\lambda_2} \left[1 - E(\lambda_i) / aP(\lambda_i, T)\right]^2 / u_{rel}^2\left(E(\lambda_i)\right) \qquad (10)$$

(10) can be written as:

$$\sum_{\lambda_i=\lambda_1}^{\lambda_2} \left(\frac{E(\lambda_i)}{aP(\lambda_i,T)}\right)^2 \left[E(\lambda_i) - aP(\lambda_i, T)\right]^2 / u^2\left(E(\lambda_i)\right) \qquad (11)$$

Since the values of the fraction in the first factor in (11) will tend to 1 due to the minimization process, (10) will provide results similar to weighted least squares (9).

Experimental Results

A spectrometer calibrated at the above mentioned detector-based FEL-lamp was employed to obtain the spectrum of an incandescent lamp (Wi41/G). The distribution temperature of the lamp should be close to 2856 K (CIE A). The wavelength interval was about 5 nm. The responsivity of the spectrometer was rather low in the blue region of the wavelength range from 400 nm to 750 nm. This led to increasing relative uncertainties of spectral irradiance in the region from 500 nm down to 400 nm (Fig. 1):

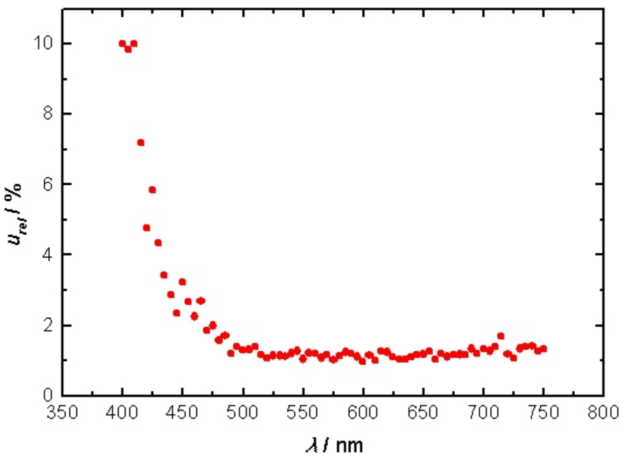

Figure 1. Relative uncertainties of the obtained spectral irradiance values of a Wi41/G incandescent lamp at approximately 2856 K (CIE A).

Since the deviations of the measured values from the curve fit obtained by CIE estimation method (8) did not exceed 5%, the concept of distribution temperature is applicable according to [1]. But the crucial condition of constant u_{rel} and constant variance of η, respectively, is obviously not fulfilled. The estimated value of T_D by minimizing (8) is unreasonable low ($T_{D,CIE}$ = 2837.6 K), while the estimator obtained by weighted least squares (9) seems plausible ($T_{D,LS}$ = 2855.0 K). T_D obtained with the weighted CIE method (10) is 2855.5 K.

In spectral regions with relative low values of spectral irradiance (blue) the CIE estimation method (8) tends to reduce the deviations from the fit curve more than in regions with high values (red). This is due to the design of this estimation method, but only desirable, if u_{rel} of spectral irradiance remains constant or approximately constant over the considered wavelength region. However, in most spectroradiometric systems, u_{rel} is greater at low wavelength (400 nm to 450 nm) than it is in most of the visible spectrum [5]. In these cases weighted least squares (9) or the weighted CIE estimation method (10) will provide more reasonable results for T_D than the estimation method currently recommended by CIE (8). Further experimental results will be presented and discussed.

Conclusions

The irradiance spectrum shape of an incandescent lamp always shows deviations to the corresponding blackbody spectrum. This is mainly due to the fact, that a tungsten filament is no genuine blackbody radiator and also due to the transmittance of the glass bulb envelope surrounding the filament [3, 6]. Thus, it seems inappropriate to denote any of the discussed estimators as “best estimator”.

From a statistical point of view all available information should be used for parameter estimation. Therefore, weighted least squares seem more feasible to determine distribution temperature, provided that the statistical information about the uncertainties of spectral irradiance data is valid.

A fundamental principle of metrology is that a measurand should be defined in such a way that its value does not depend on the method of measurement [5]. The use of weighted least squares as definition of distribution temperature would contradict this principle.

A possible solution for this problem might be to leave the CIE definition at its present form. But it should be explicitly pointed out, that this estimation method will lead to feasible results only, if the relative uncertainties of spectral irradiance data over the chosen wavelength region can be presumed constant. Otherwise weighted least squares, or the CIE estimation method weighted with u_{rel} of spectral irradiance data are to be used. The later seems more convenient and will provide similar results as weighted least squares. This would improve the comparability of distribution temperature data regarding for instance comparisons or traceability.

References

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