Simplified diffraction effects for laboratory and celestial thermal sources

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Abstract. We discuss how evaluating the diffraction effects on total power in blackbody calibrations and total-solar-irradiance measurements can be done more efficiently. The methodology that is outlined has been applied to the VIRGO and SOVIM PMO6 radiometers, the DIARAD radiometer, and the SORCE TIM radiometer.

Introduction

Diffraction affects the flow of electromagnetic radiation through optical systems, because light is a wave phenomenon. More or less radiation emitted by a source reaches a detector than the amount expected from geometrical optics. Detailed analysis of this effect is necessary in the most careful studies of optical systems, especially at longer wavelengths. People have devoted much attention to this and established a significant impact on practical radiometry. Rayleigh and Lommel analyzed diffraction effects on the intensity distribution in the detector plane for Fraunhofer and Fresnel diffraction. Wolf derived an expression for the encircled spectral power by integrating Lommel's distribution over a circular area of the detector plane for a point source, and Focke analyzed the asymptotic properties of Wolf's result. Blevin extended diffraction analysis to thermal sources and promoted the concept of the effective wavelength λ_e. At this wavelength, diffraction effects on spectral power typify diffraction effects on total power for the complex radiation of interest. Steel *et al.* considered effects of having an extended source, and Boivin studied that and diffraction effects in the form of gains in the total flux reaching an overfilled detector in the case of a non-limiting aperture. Shirley (1998) and Edwards and McCall also considered extended sources, showing how calculating diffraction effects for a point source is easily generalized to the case of an extended source.

The above work mainly considered monochromatic radiation, even if within the effective-wavelength approximation. In at least two important applications, monitoring total solar irradiance and blackbody total-power measurements, one wants to know how diffraction affects propagation of (complex) Planck radiation and total power reaching the detector. In two earlier works, Shirley (2001) and Shirley (2004), the author generalized Wolf's result to Planck radiation for Fraunhofer diffraction losses and Fresnel diffraction losses and gains, respectively. The latter work presents a means to analyze diffraction effects for a point source in terms of a double numerical integral at low source temperature and an infinite sum of divergent asymptotic expansions at high source temperature. Either analysis can be used at intermediate temperatures, but both analyses have key disadvantages, in the form of extensive computation at low temperature and the need to evaluate a complicated double sum at high temperature.

This work addresses both of the above difficulties. The numerical integration is greatly simplified except at very high source temperatures, and the lowest-order asymptotic terms have been regrouped to eliminate one summation. The simplifications have been demonstrated in the treatment of diffraction effects for the VIRGO and SOVIM PMO6 radiometers and the DIARAD radiometer, all of which are affected by diffraction at a non-limiting, view-limiting aperture, as well as the SORCE TIM radiometer, which has a limiting aperture.

Discussion

Diffraction effects depend on wavelength λ and must be treated accordingly when considering effects on spectral or total power reaching the detector in a given optical system. Spectral power $\Phi_\lambda(\lambda)$ at the detector is related to source spectral radiance $L_\lambda(\lambda)$ by $\Phi_\lambda(\lambda) = F(\lambda) G L_\lambda(\lambda)$, where G is the geometrical throughput of an optical system. The factor $F(\lambda)$ accounts for diffraction effects and is taken as unity in geometrical optics. A Planck source has

$$L_\lambda(\lambda) = \varepsilon c_1 / (\pi \lambda^5 \{\exp[c_2/(\lambda T)] - 1\}) ,$$

where ε, c_1 and c_2 are the source emissivity and radiation constants and T is the source temperature. For the total flux this gives

$$\Phi_0 = 6G\zeta(4)\varepsilon c_1 T^4 / (\pi c_2^4) = \varepsilon G \sigma_M T^4 / \pi ,$$

according to the Stefan-Boltzmann Law, where σ_M is the Stefan-Boltzmann constant.

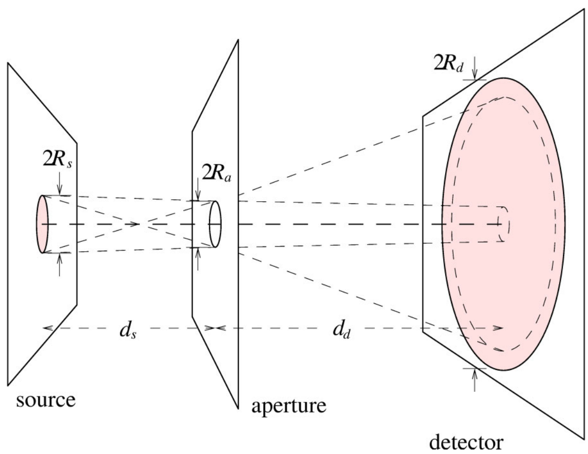

Figure 1. Generic geometry for considering diffraction effects for an extended source. It is specified by the five parameters shown and aperture focal length *f*. If the detector perimeter lies outside the larger dashed circle (as shown), the aperture is limiting. If the detector perimeter lay within the smaller dashed circle, the aperture would not be limiting.

The source radiance and power reaching the detector may be related in many systems as follows. Consider the optical system illustrated in Fig. 1, which is specified by source, aperture and detector radii, R_s, R_a and R_d, distances d_s and d_d, and aperture focal length (if it is a powered optic) *f*. As examples, these may correspond to blackbody or solar dimensions for the source, and to defining (or view-limiting) aperture and detector entrance dimensions for the optical instrumentation, with distances chosen accordingly. With $k = 2\pi/\lambda$, one may introduce the parameters

$$v_s = kR_sR_a/d_s$$
$$v_d = kR_dR_a/d_d$$
$$u = kR_a^2\,|1/d_s + 1/d_d - 1/f\,|$$
$$v_0 = \max(v_s, v_d)$$
$$\sigma = \min(v_s, v_d)/v_0$$
$$D = 4\pi^3 R_a^4 R_s^2 R_d^2/(d_s d_d \lambda v_0)^2$$

and the function

$$g(x) = \{(1-x^2)[(2+\sigma x)^2 - \sigma^2]\}^{1/2}/(1+\sigma x).$$

In paraxial systems, the spectral power reaching the detector and source spectral radiance are then related by

$$\Phi_\lambda(\lambda) = D\int_{-1}^{1} dx\, g(x) L(u,v) L_\lambda(\lambda).$$

Here we abbreviate $v = v_0(1+\sigma x)$, and $L(u,v)$ is Wolf's result for a point source. This integral allows one to deduce diffraction effects in the case of an extended source from Wolf's result. Note that many quantities, such as σ, D, $g(x)$, or u/v, as well as conditions such as $u < v$, do not depend on λ.

Extension to thermal sources

For a given value of x, and a given value of λ, one can introduce $\alpha = \lambda v$, $w = \min(u,v)/\max(u,v)$, and $q = \varepsilon c_1/(\pi\alpha^4)$, which also do not depend on λ. Similarly, for a given value of x, and a given value of T, one can introduce $A = c_2/(\alpha T)$. Suppose that one defines

$$Q(T) = q^{-1}\int_0^\infty d\lambda\, L(u,v) L_\lambda(\lambda),$$

and the constant $Q_0 = 6\zeta(4)/A^4$. This integral allows one to deduce diffraction effects on total power reaching the detector for a small thermal source from Wolf's result. Then for $u < v$ one has

$$Q(T) = Q_0 - F_B(A,w),$$

and for $u > v$ one has

$$Q(T) = w^2[Q_0 + F_B(A,w)] - F_X(A,w).$$

The functions $F_B(A,w)$ and $F_X(A,w)$ are defined in Shirley (2004) and are closely related to various contributions to Wolf's function, $L(u,v)$. Omitting these functions in the above expressions suppresses diffraction effects. Correspondingly, the fractional change of $Q(T)$ because of their inclusion yields diffraction effects on the total power reaching the detector. Once $Q(T)$ is known, it may be integrated over x in the case of an extended thermal source to obtain the expected total power reaching the detector, with diffraction effects taken into account:

$$\Phi(T) = Dq\int_{-1}^{1} dx\, g(x) Q(T).$$

Here the dependence of $Q(T)$ on x is implicit.

The chief difficulties arise when calculating $F_B(A,w)$. Its asymptotic expansion in ascending powers of A and $\log_e A$ has two disadvantages. First, because it is asymptotic, it can diverge if enough terms are included and must therefore be used with care. Second, each term involves an infinite summation over powers of w. We have found a modification of Barnes' integral representation of generalized hypergeometric functions that allows a simple, exact evaluation of $F_B(A,w)$, with very few integration points. In the case of very small A, we have also found that one can analytically sum the terms involving the lowest powers of A over all powers of w, obtaining a useful analytical expression.

We have used these innovations to analyze diffraction effects on blackbody calibrations and performance of the radiometers already mentioned. A detailed report of the innovations and application to studying the radiometers is in preparation and will be presented at the conference.

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