NPL National Physical Laboratory

The CCPR K1-a Key Comparison of Spectral Irradiance 250 nm – 2500 nm: Measurements, Analysis and Results

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Peter M. Harris, Neil J. Harrison

Presentation to: Newrad 2005

19th October 2005

Participants

Acronym	Organisation	Country
NPL	National Physical Laboratory	United Kingdom
BNM-INM	Bureau National de Métrologie - Institut National de Métrologie	France
CENAM	Centro Nacional de Metrología	Mexico
CSIRO	Commonwealth Scientific & Industrial Research Organisation	Australia
HUT	Helsinki University of Technology	Finland
IFA-CSIC	Instituto de Física Aplicada (Consejo Superior de Investigaciones Científicas)	Spain
MSL-IRL	Measurement Standards Laboratory of New Zealand - Industrial Research Limited	New Zealand
NIM	National Institute of Metrology	China
NIST	National Institute of Standards and Technology	United States of America
NMIJ	National Metrology Institute of Japan	Japan
NRC	National Research Council	Canada
PTB	Physikalisch-Technische Bundesanstalt	Germany
VNIIOFI	All-Russian Research Institute for Optical-Physical Measurements	Russian Federation

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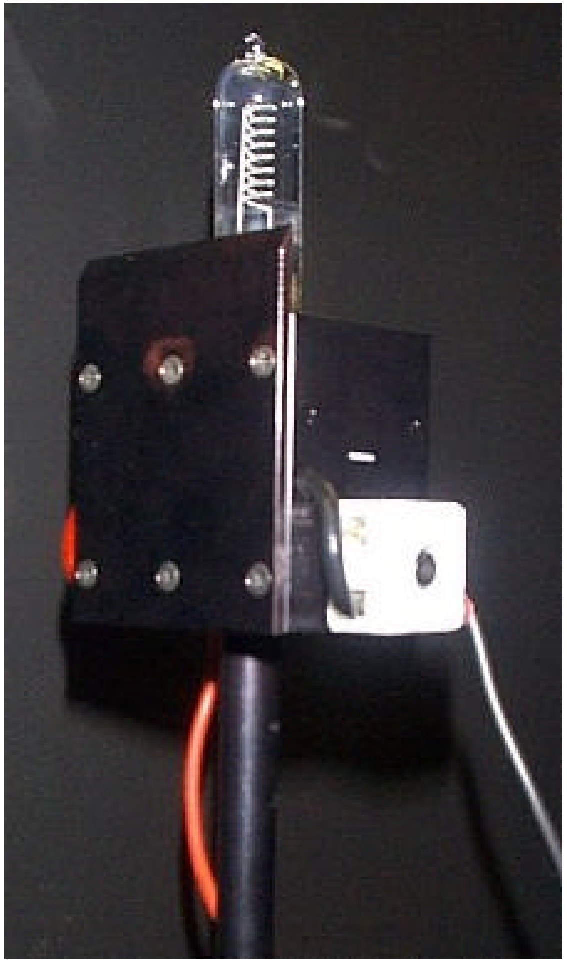
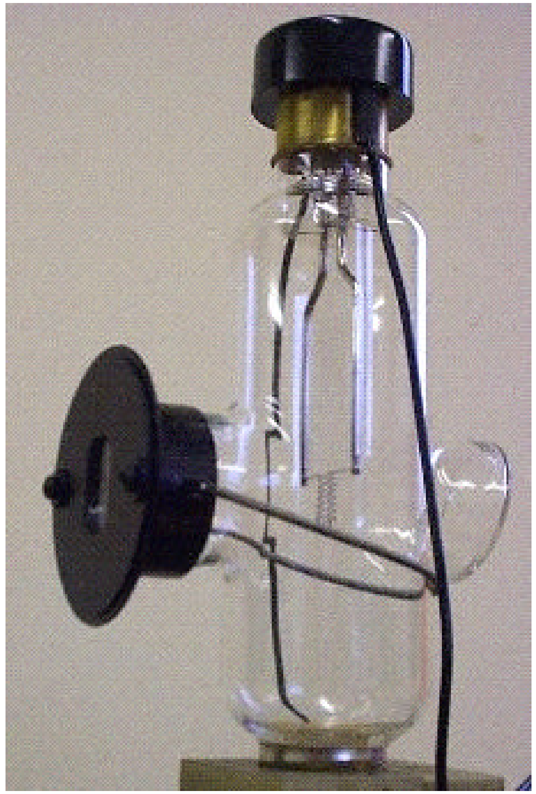
Comparison Artefacts



Type I



Type II

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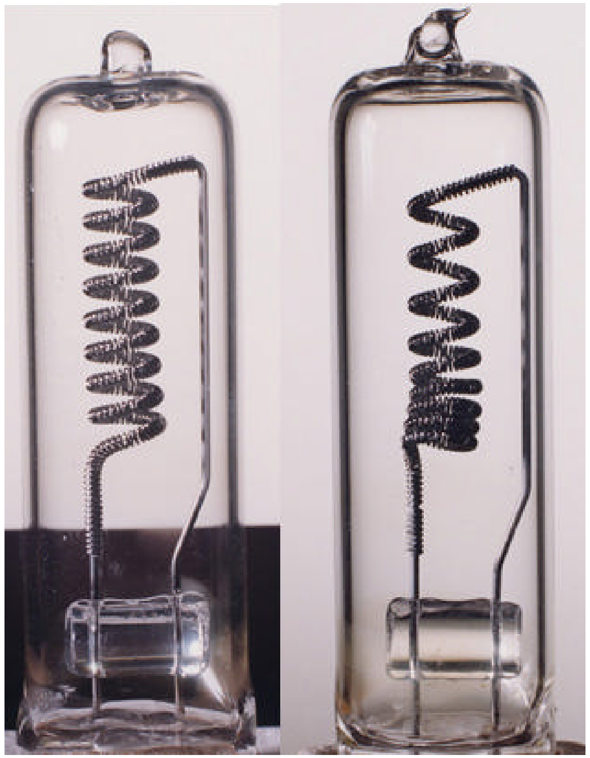
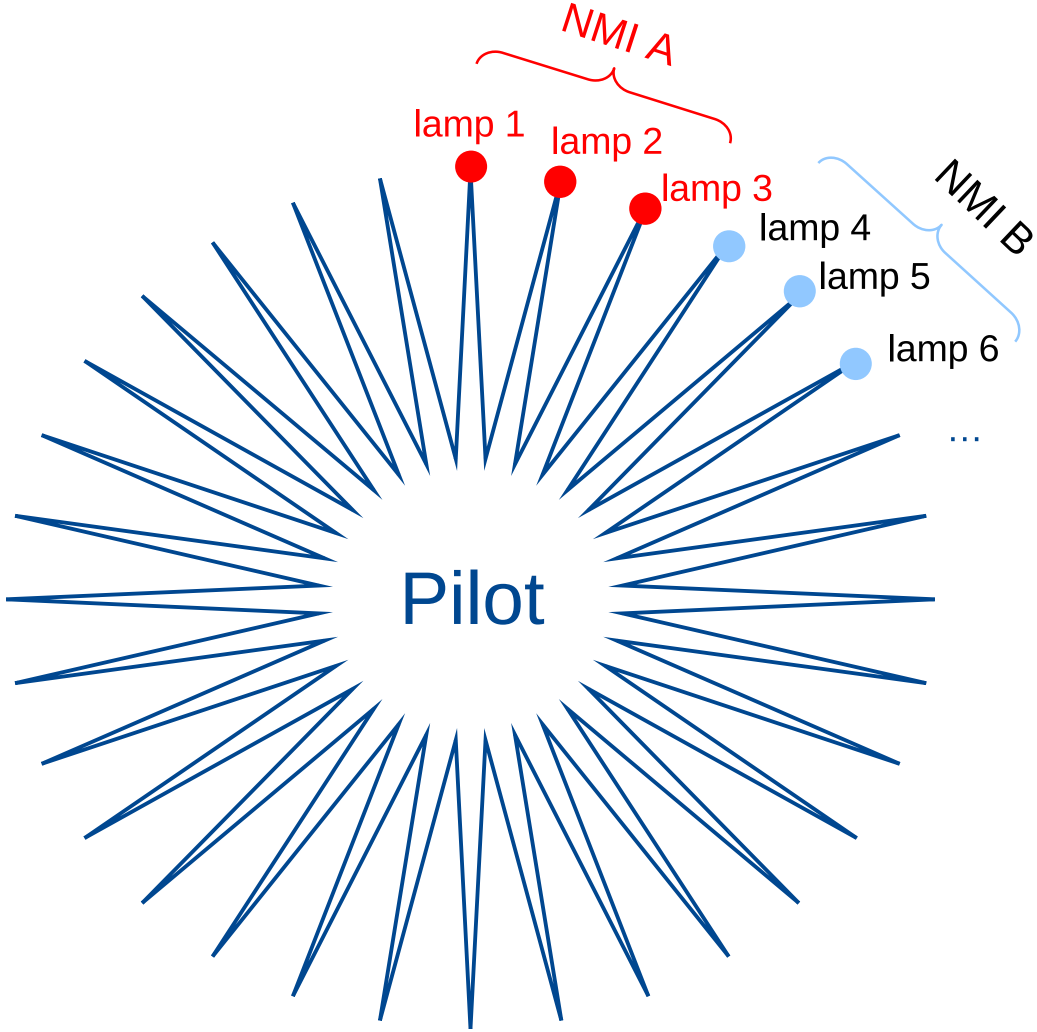
Comparison Design





Comparison design continued

- Star comparison
  - 3 lamps to each participant
  - Each participant obtained unique set of lamps
- Two measurement rounds
- Measurements
  - Pilot – Participant – Pilot - Participant
  - Participant – Pilot – Participant – Pilot

AIM:

To ensure that for every participant there was at least one lamp that survived at least two consecutive transportations:

Pilot – Participant – Pilot or Participant – Pilot - Participant

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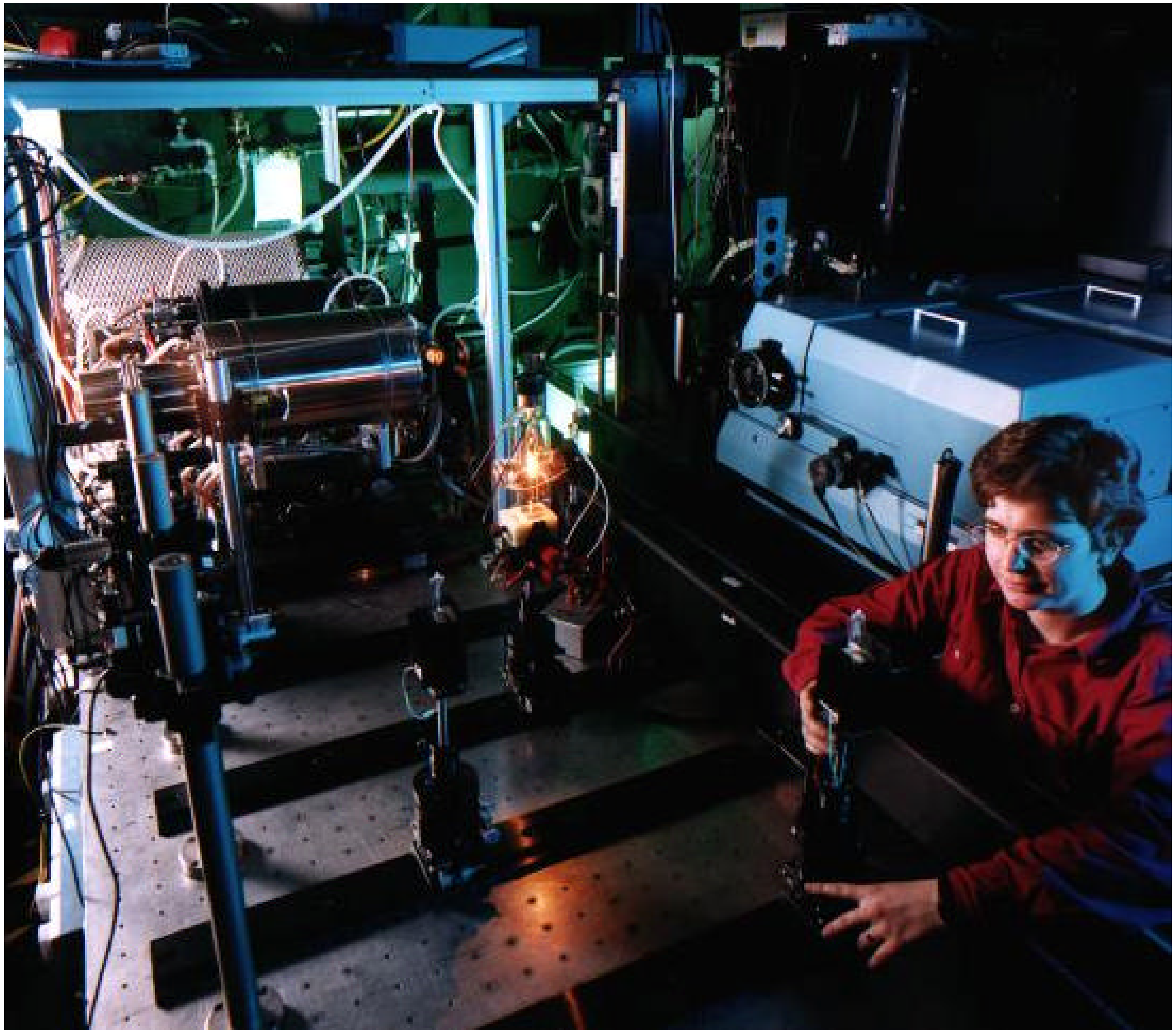
Measurements at NPL

- All lamps calibrated against NPL's primary spectral irradiance scale
- Using the SRIPS facility
- Against the ultra-high temperature blackbody



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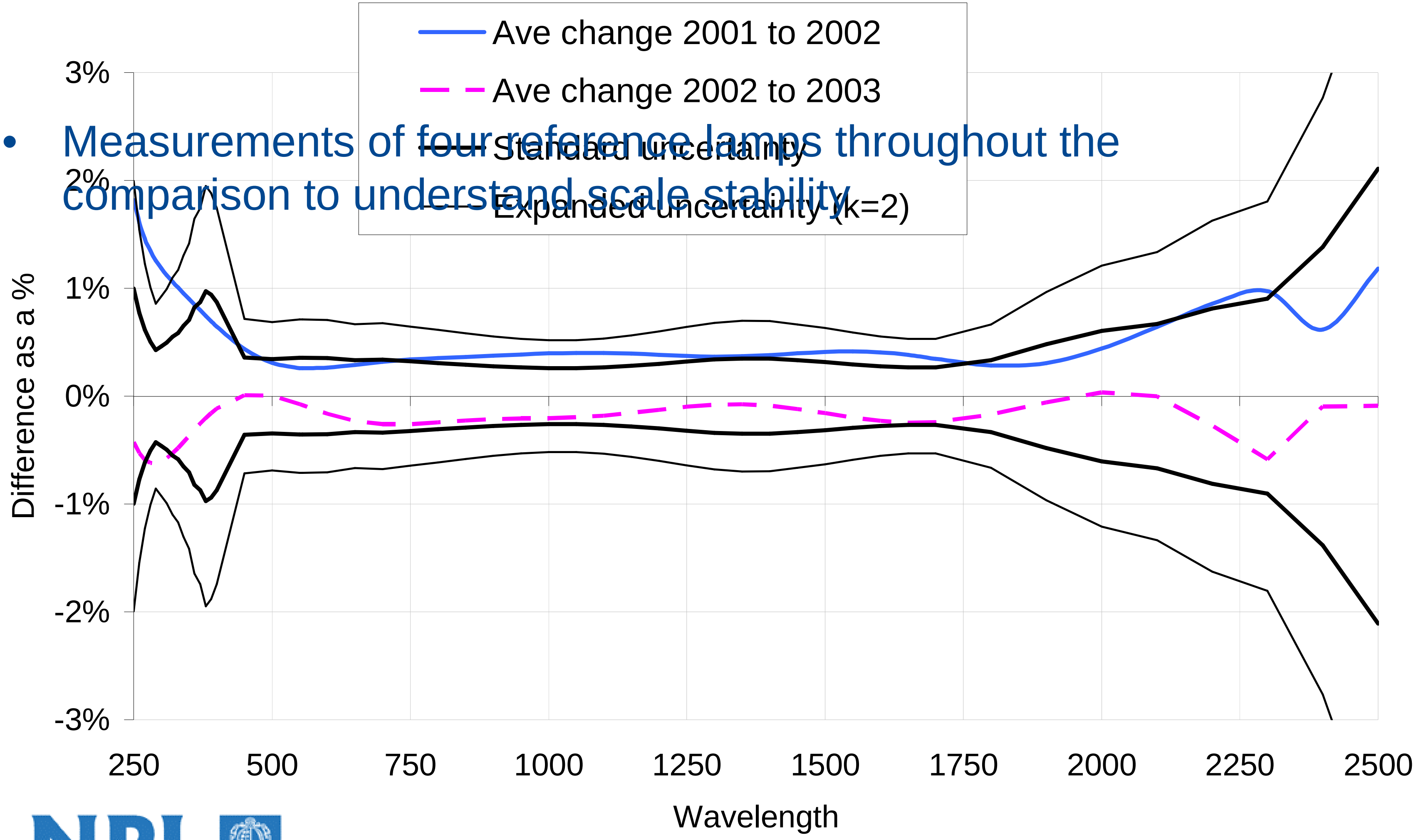
NPL Scale Stability

- NPL's scale formally established 2002
- 2001 data normalised to 2002 data
- Measurements of four reference lamps throughout the comparison to understand scale stability



Measurements at other participants

- Measurements made directly or via transfer standards to primary spectral irradiance scale
- Some use blackbody source
  - Temperature determined by ITS-90
  - Temperature determined by spectral radiometry
- Some use filter radiometers
  - To obtain an absolute point on relative spectral measurements
  - To obtain spectral information
- Traceability to other NMIs
  - MSL-IRL to NIST
  - CENAM to PTB
  - Part of NRC to “World mean of 1975”

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Peer review

- Each participant supplied pilot
  - document describing measurements
  - uncertainty statement
  - breakdown of uncertainties into those associated with
    - Systematic effects
    - Random effects
    - Round effects
- These were discussed amongst participants prior to results being analysed
- This process ensured that the data used in the analysis were as reliable and consistent as possible

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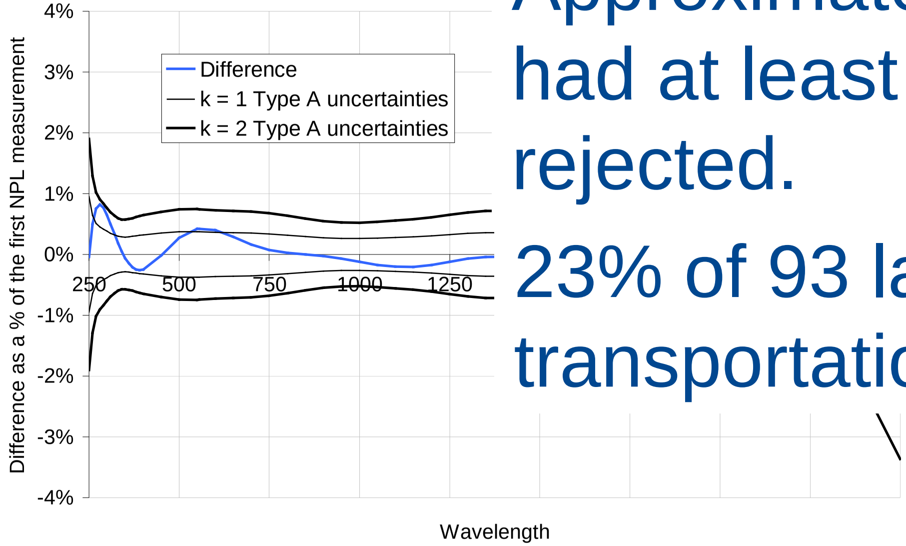
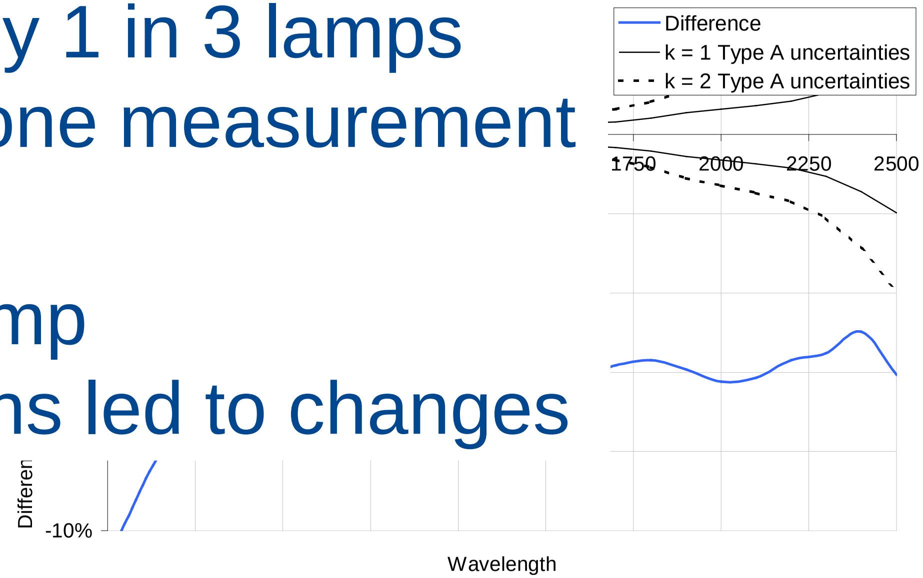
Data selection

- Prior to analysis, pilot discussed with participants which lamp measurements were to be used
- Based on repeatability of subsequent measurements by pilot or by participant



Approximately 1 in 3 lamps had at least one measurement rejected.

23% of 93 lamp transportations led to changes



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Analysis of CCPR K1-a

- Introduction of a model to describe the measurements of a lamp by a participant

$$e_{\mathrm{A1,1}} = E_1 S_{\mathrm{A}} \varepsilon_{\mathrm{A1,1}}$$

$$e_{\mathrm{p1,1}} = E_1 S_{\mathrm{p}} \varepsilon_{\mathrm{p1,1}}$$

- Systematic factors, *S*, correspond to ‘biases’ in individual participant measurements
- We need to determine these systematic factors
- To do this we use Linear Least Squares Analysis

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Linear Least Squares Analysis

- Model - multiplicative

$$e_{\mathrm{A1,1}} = E_1 S_{\mathrm{A}} \varepsilon_{\mathrm{A1,1}}$$

$$e_{\mathrm{p1,1}} = E_1 S_{\mathrm{p}} \varepsilon_{\mathrm{p1,1}}$$

- Convert to additive to make the problem linear

$$f_{\mathrm{A1,1}} = \ln e_{\mathrm{A1,1}}$$

$$F_i = \ln E_i \qquad T_n = \ln S_n \qquad \eta_i = \ln \varepsilon_i$$

- Now

$$f_{ni,r} = F_i + T_n + \eta_{ni,r}$$

Analysis of comparison: Simple example

$$f_{ni,r} = F_i + T_n + \eta_{ni,r}$$

- 4 lamps measured by Pilot, Participant A, Participant B

$$f_{\mathrm{A1}} = F_1 + T_{\mathrm{A}} \qquad e_{\mathrm{B3}} = F_3 + T_{\mathrm{B}}$$

$$f_{\mathrm{p1}} = F_1 + T_{\mathrm{p}} \qquad e_{\mathrm{p3}} = F_3 + T_{\mathrm{p}}$$

$$f_{\mathrm{A2}} = F_2 + T_{\mathrm{A}} \qquad e_{\mathrm{B4}} = F_4 + T_{\mathrm{B}}$$

$$f_{\mathrm{p2}} = F_2 + T_{\mathrm{p}} \qquad e_{\mathrm{p4}} = F_4 + T_{\mathrm{p}}$$

- We need a constraint equation – equivalent to KCRV

$$w_{\mathrm{A}} T_{\mathrm{A}} + w_{\mathrm{B}} T_{\mathrm{B}} + w_{\mathrm{p}} T_{\mathrm{p}} = 0$$

$$T_{\mathrm{A}} = \frac{-w_{\mathrm{B}}}{w_{\mathrm{A}}} T_{\mathrm{B}} + \frac{-w_{\mathrm{p}}}{w_{\mathrm{A}}} T_{\mathrm{p}}$$

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Simple example (continued)

$$
\begin{aligned}
f_{A1} &= F_1 + 0 + 0 + 0 + \frac{-w_B}{w_A} T_B + \frac{-w_p}{w_A} T_p \\
f_{A2} &= 0 + F_2 + 0 + 0 + \frac{-w_B}{w_A} T_B + \frac{-w_p}{w_A} T_p \\
f_{B3} &= 0 + 0 + F_3 + 0 + T_B + 0 \\
f_{B4} &= 0 + 0 + 0 + F_4 + T_B + 0 \\
f_{p1} &= F_1 + 0 + 0 + 0 + 0 + T_p \\
f_{p2} &= 0 + F_2 + 0 + 0 + 0 + T_p \\
f_{p3} &= 0 + 0 + F_3 + 0 + 0 + T_p \\
f_{p4} &= 0 + 0 + 0 + F_4 + 0 + T_p
\end{aligned}
$$

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Simple example (continued)

$$
\begin{aligned}
f_{A1} &= F_1 + 0 + 0 + 0 + \tfrac{-w_B}{w_A} T_B + \tfrac{-w_p}{w_A} T_p \\
f_{A2} &= 0 + F_2 + 0 + 0 + \tfrac{-w_B}{w_A} T_B + \tfrac{-w_p}{w_A} T_p \\
f_{B3} &= 0 + 0 + F_3 + 0 + T_B + 0 \\
f_{B4} &= 0 + 0 + 0 + F_4 + T_B + 0 \\
f_{p1} &= F_1 + 0 + 0 + 0 + 0 + T_p \\
f_{p2} &= 0 + F_2 + 0 + 0 + 0 + T_p \\
f_{p3} &= 0 + 0 + F_3 + 0 + 0 + T_p \\
f_{p4} &= 0 + 0 + 0 + F_4 + 0 + T_p
\end{aligned}
$$

- Solved using least squares analysis

$$\left(\mathbf{A}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{A}\right)\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{y}$$

$$
\mathbf{A} = \begin{pmatrix}
1 & 0 & 0 & 0 & \frac{-w_B}{w_A} & \frac{-w_p}{w_A} \\
0 & 1 & 0 & 0 & \frac{-w_B}{w_A} & \frac{-w_p}{w_A} \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1
\end{pmatrix}, \quad
\mathbf{x} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ T_B \\ T_p \end{pmatrix}, \quad
\mathbf{y} = \begin{pmatrix} f_{A1} \\ f_{A2} \\ f_{B3} \\ f_{B4} \\ f_{p1} \\ f_{p2} \\ f_{p3} \\ f_{p4} \end{pmatrix}. \quad
\mathbf{V} = \begin{pmatrix}
u^2(f_{A1}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & u^2(f_{A2}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & u^2(f_{B3}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & u^2(f_{B4}) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & u^2(f_{p1}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & u^2(f_{p2}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & u^2(f_{p3}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(f_{p4})
\end{pmatrix}
$$

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Analysis of CCPR K1-a

- Model - multiplicative

$$e_{\mathrm{A1,1}} = E_1 S_{\mathrm{A}}$$

$$e_{\mathrm{p1,1}} = E_1 S_{\mathrm{p}}$$

- Convert to additive

$$f_{\mathrm{A1,1}} = \ln e_{\mathrm{A1,1}}$$

$$F_i = \ln E_i$$

$$T_n = \ln S_n$$

- Constraint equation

$$\sum w_i T_i = 0$$

$$\prod S_i^{\omega_i} = 1$$

- Solution by least squares analysis

$$\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{y}$$

$$\mathbf{x} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ T_{\mathrm{B}} \\ T_{\mathrm{C}} \\ \vdots \\ T_{\mathrm{p}} \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} f_{\mathrm{A1,1}} \\ f_{\mathrm{A1,2}} \\ f_{\mathrm{A2,1}} \\ f_{\mathrm{A2,2}} \\ f_{\mathrm{B3,1}} \\ \vdots \\ e_{\mathrm{p1,1}} \\ e_{\mathrm{p1,2}} \\ e_{\mathrm{p2,1}} \\ \vdots \end{pmatrix}$$

A : Design matrix – 0s and 1s to say which participant measured which lamp

V : covariance matrix

- Convert back to multiplicative model

$$S_n = e^{T_n}$$

Why did we choose this model?

- Uses “text book” mathematics
  - Least squares analysis: 200th anniversary this year!
  - Algorithms are available
  - Well understood
- Does all the “averaging” in one step
- Systematic effects explicitly stated in model
- Random effects – uncertainties can be tested by chi-squared tests
- Straightforward to deal with complexity of a comparison that involved so many measurements over so long and partial correlations

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Results of the CCPR K1-a Key Comparison

- Unfortunately the Draft B report has not been approved by the CCPR Key Comparison Working Group
- Once approval has been obtained – results will be accessible on BIPM website: http://kcdb.bipm.org/
- So I can’t show you results relative to the KCRV
- But – I can show you results relative to the pilot

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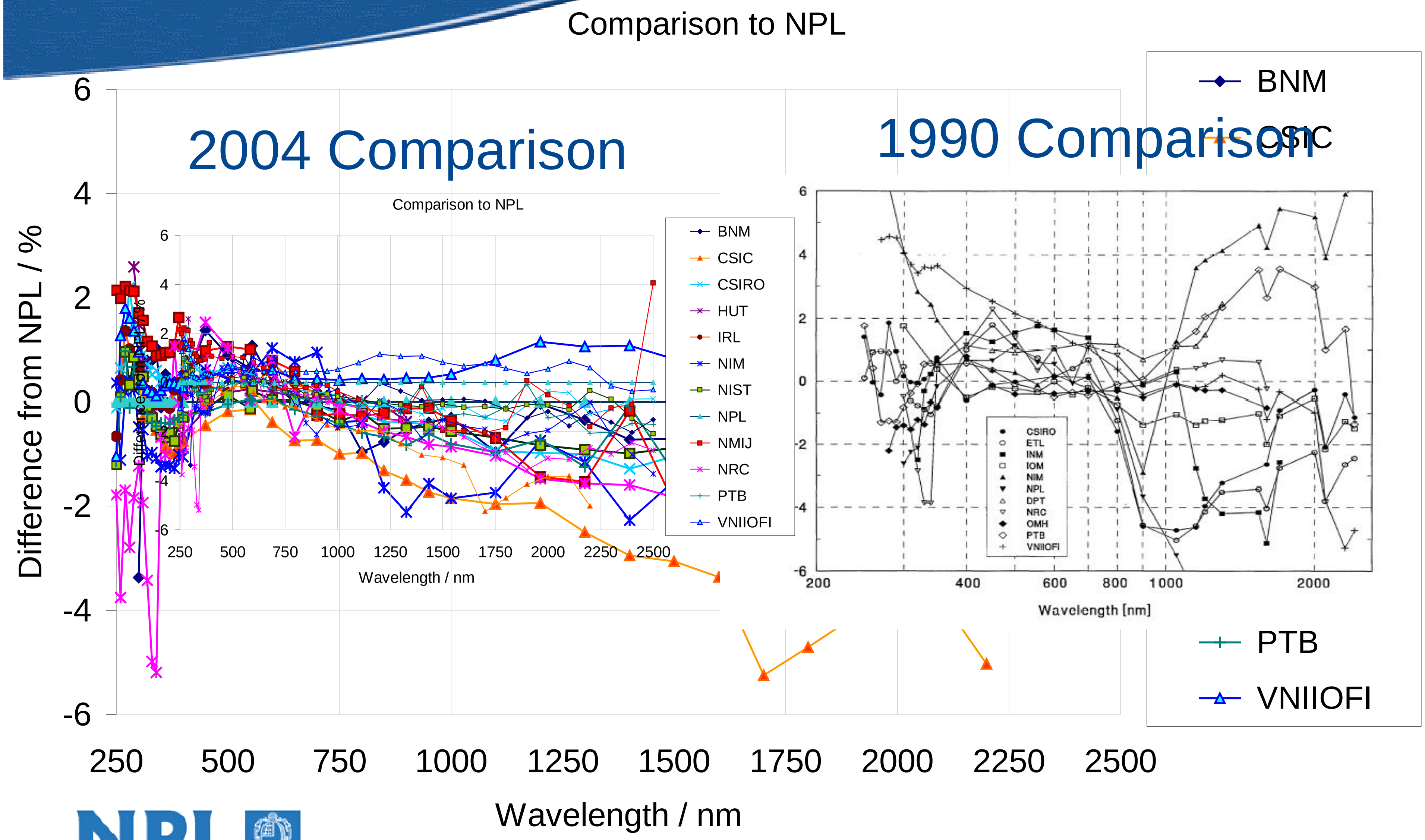
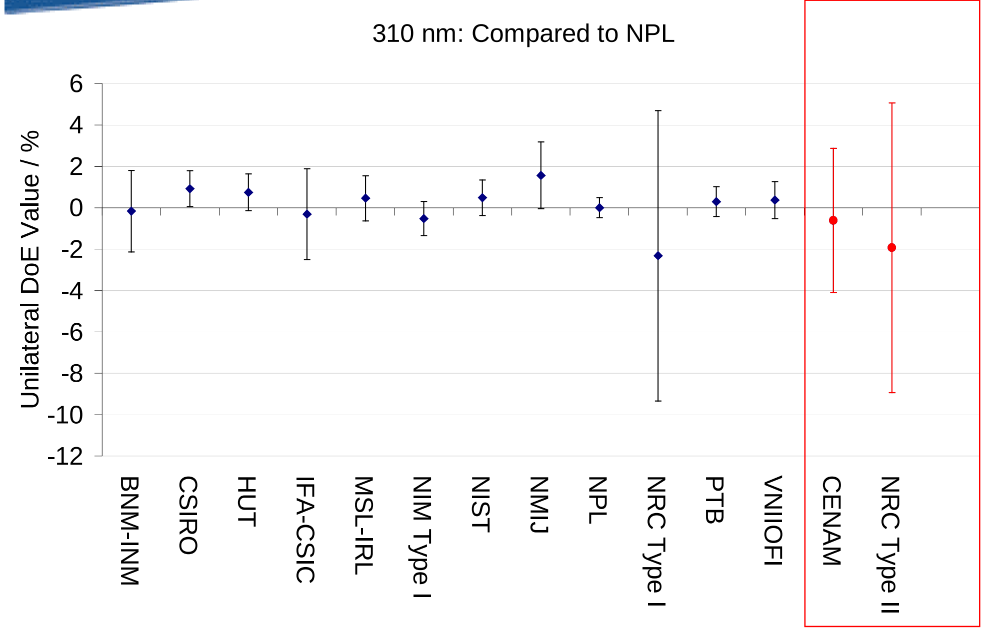
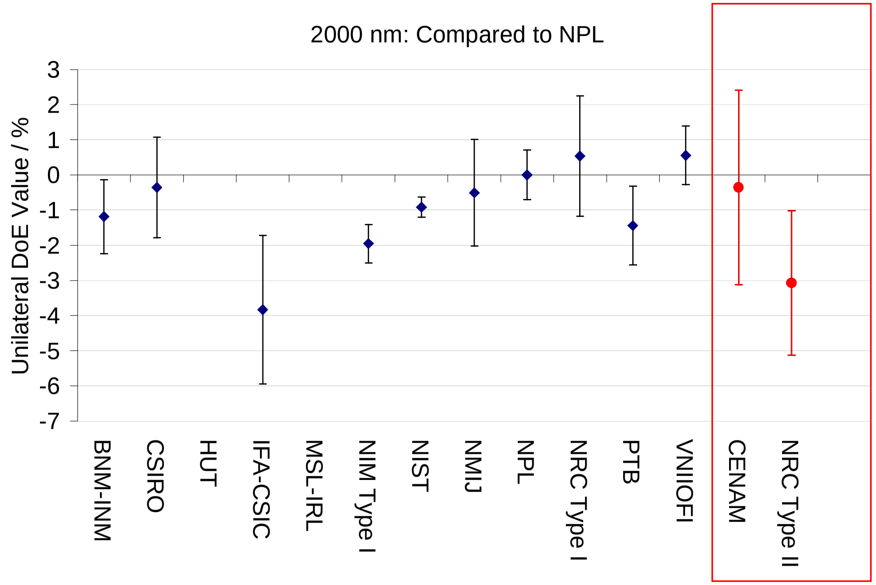
Results relative to NPL



Results compared to NPL: 310 nm and 2000 nm





Conclusions

- Results of comparison are significantly better than those of previous comparisons
- Blind peer-review of techniques and uncertainty statements helped ensure data reliability prior to comparison
- Analysis methods developed to cope with complexity
- Transfer standards did change during transportation – and perhaps limited accuracy of comparison

- And finally…

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How many people does it take to compare 57 light bulbs?

- Since 1999 the following staff have worked on the organisation, measurements, data analysis, checking, proof reading, …
- Emma Woolliams, Nigel Fox, Neil Harrison, Maurice Cox, Peter Harris, David Pollard, Leon Rogers, Ruth Montgomery, Heather Pegrum, Andy Sibley, Paul Kenward, Ian Smith, Teresa Hunt, Subrena Harris, Boris Khlevnoy, Diane Beauvais, Stuart Windsor, Will Servantes, Stuart Prince, Malcolm White, Peter Clarke, Michael Munteanu, Peter Woolliams, Ray Lambe, Andrew Hanson, Neil Walker, Andrew Brown, Jian Wang, Bill Hartree, Jenny Carter, Clare Poppett, John Mountford, Tim Burnitt, Rainer Winkler

- In addition we have had direct contact with the following people in other organisations:
- Bernard Rougie, Eric Rosas, Carlos Matamoros, Wolfgang Schmid, Mark Ballico, Frank Wilkinson, Petri Kärhä, Erkki Ikonnen, Alicia Pons, Kathryn Nield, Dai Caihong, Howard Yoon, Charles Gibson, Yoshi Ohno, Al Parr, Tatsuya Zama, Arnold Gaertner, Peter Sperfeld, Boris Khlevnoy,Victor Sapritsky …

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